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STRESS-STRAIN STATE OF ROCK MASSIFS AROUND TUNNELS LOCATED IN A DIP-LAYERED MASSIF

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Abstract: The article presents the results of a research on the stress-strain state of rock massifs around tunnels located in a dip-layered massif. As a result of the research, the authors developed a technique for determining stress fields around workings associated with the impact of water pressure in an anisotropic massif. The study was conducted using Muskhelishvili's method, which made it possible to calculate the components of stresses and relative deformations with account of the relief, gravitational and tectonic forces, as well as the hydrostatic pressure of water. The results show that the distribution of stresses around the tunnel changes insignificantly. The stresses extend in the arch and footing, while in other areas they are compressed. High accuracy of calculations is confirmed by the permissible error. Contour stresses caused by hydrostatic pressure are of a tensile nature and can threaten less durable rocks, with a pressure of 20 to 200 MPa. The vaulted section of the tunnel with vertical walls effectively distributes loads, especially under soil and seismic impacts. When selecting the types and strength of fastening, it is important to take into account the values of water pressure and maximum tensile contour stresses. The stress-strain state of tunnels and workings is analysed with the use of the MATCHAD software environment.

The vaulted section of the tunnel with vertical walls effectively distributes loads, especially under soil and seismic impacts. When selecting the types and strength of fastening, it is important to take into account the values of water pressure and maximum tensile contour stresses. The analysis of the stress-strain state of tunnels and working using the MATCHAD software environment is presented. New patterns of stress and strain distribution are established, which is important for the design and operation of transport structures. The results can be used to develop recommendations for tunnel design taking into account the forces in the massif.

Keywords: tunnel, mine working, stresses, deformation, gravitational force, seismic force, anisotropy, hydrostatic pressure.

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НАПРЯЖЕННО-ДЕФОРМИРОВАННОЕ СОСТОЯНИЕ МАССИВОВ ПОРОД ВОКРУГ ТУННЕЛЕЙ, РАСПОЛОЖЕННЫХ В НАКЛОННО-СЛОИСТОМ МАССИВЕ

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Аннотация: В статье приведены результаты исследований напряженно-деформированного состояния массивов пород вокруг туннелей, расположенных в наклонно-слоистом массиве. В результате проведенных исследований была разработана методика определения полей напряжений вокруг горных выработок, связанные с воздействием напора воды в анизотропном массиве. Исследование проведено с применением метода Мусхелишвили, что позволило вычислить компоненты напряжений и относительных деформаций с учетом рельефа, гравитационных и тектонических сил, а также гидростатического напора воды. Результаты показывают, что распределение напряжений вокруг туннеля меняется незначительно. В своде и подошве фиксируются растяжения, в то время как в других областях наблюдается сжатие. Высокая точность расчетов подтверждается допустимой погрешностью. Контурные напряжения, вызванные гидростатическим напором, имеют растягивающий характер и могут угрожать менее прочным породам, с давлением от 20 до 200 МПа. Сводчатое сечение туннеля с вертикальными стенками эффективно распределяет нагрузки, особенно в условиях грунтовых и сейсмических воздействий. При выборе типов и прочности крепления важно учитывать величины напора воды и максимальные растягивающие контурные напряжения. Представлен анализ напряженно-деформированного состояния туннелей и горных выработок с использованием программной среды MATCHAD. Установлены новые закономерности распределения напряжений и деформаций, что имеет значение для проектирования и эксплуатации транспортных сооружений. Результаты могут быть использованы для разработки рекомендаций по проектированию туннелей с учетом действующих сил в массиве.

Ключевые слова: туннель, выработка, напряжения, деформация, гравитационая сила, сейсмическая сила, анизотропия, гидростатический напор

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Introduction

In recent years Kyrgyzstan has witnessed active infrastructure development associated with the need to improve transport accessibility and efficient use of natural resources. One of the key areas of this process is the construction of highways, in particular, the North-South highway, designed to connect different regions of the country. The implementation of this project actively involves building transport tunnels in complex mountain ranges. For example, during the construction of tunnels in the Tushashu Pass area, where a high level of seismic activity and the presence of dip-layered rocks characterize geological conditions, it is necessary to conduct a detailed analysis of the stress-strain state.

The Kumtor mine, one of the largest gold mining enterprises in the country, plans to introduce underground mining. This mining method involves the creation of complex underground structures and workings. It will require a thorough study of mining conditions. It is also important to take into account the influence of geological factors, such as the presence of aquifers and structural features of rocks, in order to minimize the risks of collapses and other accidents.

Another example worth noting is the construction of hydroelectric power plants (HPPs), where power hydraulic tunnels are created and operated. For example, an HPP project on the Naryn River requires the use of modern technologies to ensure the reliability and durability of structures. Tunnels must be designed taking into account the hydrogeological conditions of the region, which can vary significantly depending on the time of year and the level of precipitation.

Thus, all the above-mentioned aspects emphasize the urgent need for comprehensive studies of the stress-strain state of rocks in Kyrgyzstan. The solution to this issue will not only improve the safety of construction work, but will also ensure a reliable operation of underground structures. A deep understanding of geological conditions will help minimize the risks associated with collapses and other accidents, which, in turn, will contribute to the sustainable development of the country's economy in general.

Considering that over 90% of the republic's territory is occupied by mountainous areas, it is necessary to develop new methods for calculating and predicting the state of tunnels. This will not only improve the safety of their operation, but also improve the design system of hydraulic structures in a complex terrain.

Research objective: development of methodological and mathematical foundations for calculating and assessing the stress-strain state of massifs around tunnels, taking into account the influence of the location and shape of the tunnel, as well as the features of the mountainous terrain. Special attention will be paid to the combined effects of gravitational and seismic forces, as well as the hydrostatic water pressure.

Research methods

The following methods are used in the research:

1. The method of two-dimensional elasticity theory - for the analysis of the stress-strain state (SSS) of the massifs around the tunnels.
2. The apparatus of conformal mappings – for the transformation of complex geometric shapes into simpler ones, which allows to simplify the calculation problems.
3. The Kolosov-Muskhelishvili's method – for solving the problems related to plane deformations and stresses in rocks.
4. Use of MATHCAD software – for performing analytical calculations and visualization of results, performing calculations of stress fields around the tunnel with a graphical representation of data using a computer.
5. Numerical calculation – for the numerical analysis of various scenarios of impact on the massifs, including combined forces.

Principle

Near the Earth's surface, at a depth of H, a tunnel, operating in both pressured and pressureless mode, was dug in a dip-layered massif. The massif in $X'OY'$ plane is represented by a transversely isotropic elastic body. The gravitational acceleration g is directed vertically downwards, opposite to OY axis, the seismic force is directed from the depth to the Earth's surface and forms an angle δ with the axis OY.

B. Zhumabaev studies the slope of the layering with respect to the axis OX', which makes up an angle φ, while the axis OY', directed along the layering and perpendicular to it (Fig. 1), allows us to characterize the properties of the massif using five independent constants [1-3]:

E_1, E_2 - Young's modules in the plane of isotropy and perpendicular to it;

ν_1, ν_2 - Poisson's ratios in the same planes as E_1, E_2;

G_1 and G_2 - shear modules,

where $G_1 = E_1/[2(1-\nu)]$.

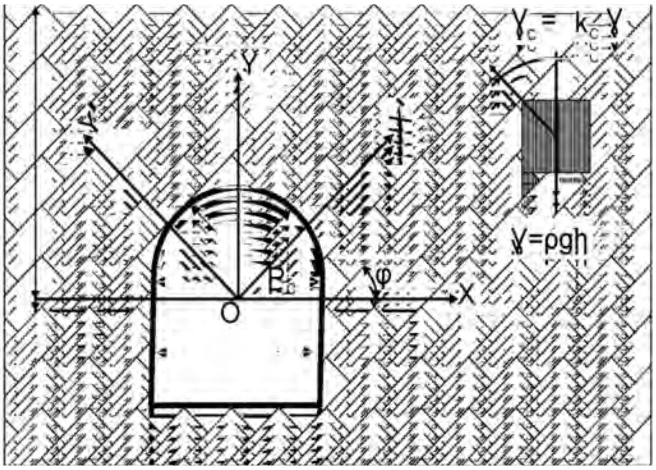

Fig. 1. Calculation diagram of a hydraulic tunnel with sloping /
Рис. 1. Расчетная схема гидротехнического туннеля, расположенного наклонно-слоистом массиве

The research of S.A. Batugin and R.K. Nirenburg studies the properties of three types of rocks: sandy shale, fine-grained granite and limestone (Table 1). Sandy shale has a pronounced anisotropy due to its bedded structure. At the same time, fine-grained granite and limestone demonstrate properties close to an isotropic body, which makes their performance in various conditions more uniform [4].

Table 1 / Таблица 1

Properties of rocks / Характеристики горных пород

Rock type / Горная порода	Elastic constants / Упругие постоянные				
	$E_1 \cdot 10^4$, MPa	$E_2 \cdot 10^4$, MPa /	$G_2 \cdot 10^4$, MPa	ν_1	ν_2
Sandy shale / Песчанистый сланец	1.07	0.52	0.12	0.41	0.20
Limestone / Известняк	4.18	3.41	1.45	0.29	0.18
Fine-grained granite / Гранит мелкозернистый	8.23	8.44	3.28	0.24	0.29

Hooke's law for *XOY* arbitrary axis can be obtained by applying a coordinate transformation $x = x'\cos\varphi - y'\sin\varphi$, $y = x'\sin\varphi + y'\cos\varphi$, associated with the rotation of $X'OY'$ plane around a given axis by an angle φ. In this case, the relationship between the stresses and strains in three-dimensional space is described as [4]:

$$\left.\begin{aligned}
\varepsilon'_x &= a_{11}\sigma'_x + a_{12}\sigma'_y + a_{13}\sigma'_z + a_{16}\tau'_{xy}\\
\varepsilon'_y &= a_{12}\sigma'_x + a_{22}\sigma'_y + a_{23}\sigma'_z + a_{26}\tau'_{xy}\\
\varepsilon'_z &= a_{13}\sigma'_x + a_{23}\sigma'_y + a_{33}\sigma'_z + a_{36}\tau'_{xy}\\
\gamma_{yz} &= a_{44}\tau'_{zy} + a_{45}\tau'_{xz}\\
\gamma_{xz} &= a_{45}\tau_{zy} + a_{55}\tau_{xz}\\
\gamma_{xy} &= a_{16}\sigma_x + a_{26}\sigma_y + a_{36}\sigma_z - a_{66}\tau_{xy}
\end{aligned}\right\} \qquad (1)$$

The coefficients a_{ij} in (1) are solved through angle φ, using the ratios of constants given in Table 1:

$$a_{11} = \frac{1}{E_1}\cos^4\varphi + \frac{1}{4}\left(\frac{1}{G_2} - \frac{2\nu_2}{E_1}\right)\sin^2\varphi + \frac{1}{E_2}\sin^4\varphi,$$

$$a_{22} = \frac{1}{E_1}\sin^4\varphi + \frac{1}{4}\left(\frac{1}{G_2} - \frac{2\nu_2}{E_1}\right)\sin^2\varphi + \frac{1}{E_2}\cos^4\varphi,$$

$$a_{33} = \frac{1}{E_1},\ \ a_{12} = \frac{1}{4}\left(\frac{2(1+\nu_1)}{E_1} + \frac{1}{E_2} - \frac{1}{G_2}\right)\sin^2 2\varphi - \frac{\nu_2}{E_1}, \qquad (2)$$

$$a_{44} = \frac{1}{G_2}\cos^2\varphi + \frac{2(1+\nu_1)}{E_1}\sin^2\varphi,\ \ a_{35} = \frac{2(1+\nu_1)}{E_1}\cos^2\varphi + \frac{1}{G_2}\sin^2\varphi,$$

$$a_{66} = \frac{1}{G_2} + \left(\frac{1+2\nu_2}{E_1} + \frac{1}{E_2} - \frac{1}{G_2}\right)\sin^2 2\varphi,\ \ a_{13} = -\frac{\nu_1}{E_1}\cos^2\varphi - \frac{\nu_2}{E_1}\sin^2\varphi,$$

$$a_{23} = -\frac{\nu_1}{E_1}\sin^2\varphi - \frac{\nu_2}{E_1}\cos^2\varphi,\ \ a_{36} = \frac{\nu_2 - \nu_1}{E_1}\sin 2\varphi,$$

$$a_{45} = \left(\frac{1+\nu_1}{E_1} - \frac{1}{2G_2}\right)\sin 2\varphi,$$

$$a_{16} = \left[\frac{1-\nu_2}{E_1}\cos^2\varphi - \left(\frac{\nu_2}{E_1} + \frac{1}{E_2}\right)\sin^2\varphi - \frac{1}{2G_2}\cos 2\varphi\right]\sin 2\varphi,$$

In this case, the stress-strain state (SSS) of the layered massif without a tunnel is described by the following formulas [4-6]:

$$\sigma_x = -\lambda_x H,\ \ \sigma_y = -\lambda_y H,\ \ \sigma_z = -\lambda_z H,\ \ \tau_{xy} = -\lambda_{xy} H \qquad (3)$$

here

$$\begin{gathered}
\lambda_x = A_1\frac{\Delta_x}{\Delta_0} + A_2\frac{\Delta'_x}{\Delta'_{xy}},\ \ \lambda_y = A_1 + A_2\frac{\Delta'_x}{\Delta'_{xy}},\ \ \lambda_z = A_1\frac{\Delta_z}{\Delta_0} + A_2\frac{\Delta'_z}{\Delta'_{xy}},\\
\lambda_{yz} = A_1\frac{\Delta_{yz}}{\Delta_0} + A_2\frac{\Delta'_{yz}}{\Delta'_{xy}},\ \ \lambda_{xz} = A_1\frac{\Delta_{xz}}{\Delta_0} + A_2\frac{\Delta'_x}{\Delta'_{xy}},\ \ \lambda_{xy} = A_1\frac{\Delta_{xy}}{\Delta_0} + A_2,
\end{gathered} \qquad (4)$$

where $A_1 = \gamma \cdot (1 - K_c\cos\delta)$, $A_2 = \gamma \cdot K_c\sin\delta$, Δ_0, Δ_x, Δ_z, Δ_{xy}, Δ'_{xy}, Δ'_x, Δ'_y, Δ'_z are expressed through ratios a_{ij} in (1).

In this case the coefficients λ_x, λ_y, λ_z, λ_{xy} for the corresponding components of the stress-strain state (SSS) depend on the elastic characteristics of the massif a_{ij}, such as Young's modulus, Poisson's ratio and other parameters that determine the deformation properties of the material [6,7]. In addition, the coefficients λ_i in (4) may depend on the geometric characteristics of the layered massif, such as the thickness of the layers, the orientation of the layers in reference to the load direction, and the presence of cracks or other defects. These factors are critical for determining the mechanical performance of the material under external loads and must be taken into account while modeling the stress-strain state (SSS) [8,9].

To increase accuracy, we will provide examples based on calculations for sandy shale. In particular, studies have shown that the mechanical properties of sandy shale, such as Young's modulus and compressive strength, vary depending on its texture and degree of cementation (Table 2).

Analysis of the coefficient values in table 2 shows [7]:

- in the absence of a seismic force ($K_c = 0$) the lateral thrust along the direction of the stratification λ'_z can vary up to 60% depending on the angle of the slope. At the same time, the lateral thrust changes over 5 times, while the shear stresses can reach 30% of the weight of the overlying rock column;

Table 2 / Таблица 2

Stress distribution in an anisotropic half-space / Распределение напряжений в анизотропном полупространстве

Degree / Град.	$K_c = 0$			$K_c = 0{,}2,\ \delta = 60^0$				$K_c = 0{,}4,\ \delta = 90^0$			
	λ'_x	λ'_z	λ'_{xy}	λ'_x	λ'_y	λ'_z	λ'_{xy}	λ'_x	λ'_y	λ'_z	λ'_{xy}
0	0.339	0.339	0.000	0.271	0.800	0.271	0.346	0.339	1.000	0.339	0.400
10	0.339	0.365	-0.030	0.669	0.762	0.450	0.332	0.803	0.956	0.548	0.370
20	0.549	0.4360	-0.003	0.829	0.797	0.545	0.344	0.999	0.997	0.662	0.397
30	0.690	0.518	0.104	0.864	0.864	0.605	0.430	1.051	1.073	0.738	0.504
40	0.703	0.564	0.239	0.785	0.939	0.634	0.537	0.960	1.161	0775	0.639
50	0.577	0.533	0.313	0.601	1.022	0.626	0.597	0.738	1.237	0.765	0.713
60	0.404	0.514	0.299	0.387	1.112	0.602	0.585	0.477	1.360	0.734	0.699
70	0.259	0.474	0.223	0.204	1.189	0.575	0.525	0.256	1.450	0.700	0.623
80	0.169	0.477	0.117	0.098	1.150	0.516	0.440	0.126	1.404	0.630	0.517
90	0.140	0.436	0.000	0.112	0.800	0.350	0.346	0.140	1.000	0.438	0.400

● the impact of seismic forces leads not only to an increase in normal stresses, but also to a significant increase in shear stresses τ_{xy}.

● the horizontal direction of seismic forces at ($K_c = 0{,}4$) leads to an increase in all stress components (σ_x, σ_y, τ_{xy}). In this regard, it is advisable to carry out predictive calculations of the strength of hydraulic tunnels taking into account the horizontal seismic impact ($\delta = 90^0$).

The tunnel is located at a depth H, and its contour L is described by parametric equations (4)

$$x = R\left[\cos\theta + \varepsilon \sum_{k=2}^{5} d_k \cos k\theta\right],\quad y = R\left[C\sin\theta + \varepsilon \sum_{k=2}^{5} d_k \sin k\theta\right] \qquad (4)$$

where $R > 0$ – characteristic linear dimension of the tunnel cross-section;

$0 \le \theta \le 360^0$ - polar angle;

ε - small parameter;

d_k - real numbers (table 3).

Here, the tunnel cross section is modeled using the mapping function [11-12]:

$$\omega(\zeta) = e^{i\delta} R\left[\zeta + \omega_0(\zeta)\right];\quad \omega_0(\zeta) = \sum_{k=2}^{5} \frac{d_k}{\zeta^k};\ \zeta = \rho e^{i\theta}.$$

Table 3 / Таблица 3

Parameters for modeling a vaulted tunnel section /
Параметры для моделирования сводчатого сечения туннеля

δ	d_1	d_2	d_3	d_4	d_5
90⁰	0.1416	0.0651	-0.0970	0.0371	0.0019

Using parametric equations (4) and by varying the values of C, ε, d_k (k = 2, 3, 4, 5) it is possible to model complicated tunnel topography accurately, which is key to subsequent analysis of its stability and strength. These equations provide a detailed description of the tunnel shape, including characteristics such as radius and length. This not only facilitates visualization of the structure, but also allows for in-depth calculations that take into account the effects of various factors, including seismic loads and geological conditions.

The stress distribution around a tunnel with a vaulted cross-section (vertical walls) can be described as follows [12,13]:

Fig. 2 shows the calculation diagram of the tunnel and its area division grid.

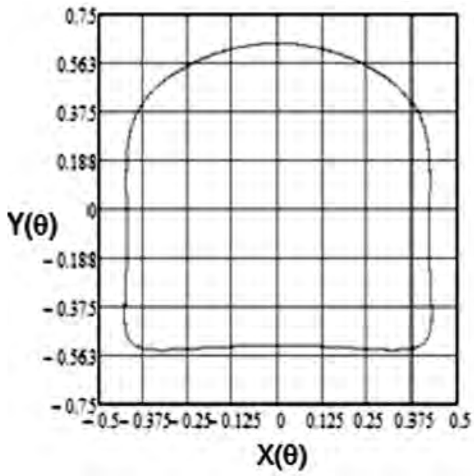

Fig. 2. Calculation diagram of the tunnel (reduced cross-section with developed vertical walls) /
Рис. 2. Расчетная схема туннеля (сводчатое сечение с развитыми вертикальными стенками)

$$\sigma_x^o = -\lambda_x H + \sigma_x + \sigma_x^H,\quad \sigma_y^o = -\lambda_y H + \sigma_y + \sigma_y^H,$$
$$\tau_{xy}^o = -\lambda_{xy} H + \tau_{xy} + \tau_{xy}^H. \qquad (5)$$

The first summands in equation (5) represent the stress-strain state of a dip-layered massif under bulk forces, including $A_1 = \gamma \cdot (1 - \cos\delta)$ - vertical and $A_2 = \gamma \cdot K_c \sin\delta$, -horizontal components.

The second summands (σ_x, σ_y, τ_{xy}) in (5) characterize the formation of a mine working with contour (4) and satisfy the boundary condition [7]:

$$X'_n = \sigma_x \cos(n,x) + \tau_{xy}\cos(n,y) = -\lambda_x H \frac{dy}{ds} + \lambda_{xy} H \frac{dx}{ds},$$
$$Y'_n = \tau_{xy}\cos(n,x) + \sigma_y \cos(n,y) = -\lambda_{yx} H \frac{dy}{ds} + \lambda_y H \frac{dx}{ds}. \qquad (6)$$

The third summand σ_x^H, σ_y^H, τ_{xy}^H in (5) determine the effect of the hydrostatic water pressure – P, acting perpendicularly at each point of the tunnel contour L in (4) and corresponds to the boundary condition:

$$X'_n = -P_0 \cos(n,x) = P_0 \frac{dy}{ds},\quad Y'_n = -P_0\cos(n,y) = -P_0 \frac{dx}{ds}, \qquad (7)$$

where d_x, d_y, d_s - differential arcs of the contour are defined as:

$$dx = \left[\sin\theta + \varepsilon \sum_{k=2}^{5} k d_k \sin k\theta\right],\quad dy = R\left[C\cos\theta - \varepsilon \sum_{k=2}^{5} k d_k \cos k\theta\right], \qquad (8)$$
$$ds^2 = (dx)^2 + (dy)^2.$$

Based on conditions (6), if external conditions are X'_n, Y'_n и X^H_n, Y^H_n, in (6) and (7), then it is possible to determine the distribution of stress fields σ_x, σ_y, τ_{xy} in an anisotropic plane with a mine working (4) [12-14]:

$$2\operatorname{Re}\left[\varphi_1(z'_1)+\varphi_2(z'_2)\right]=-\int_0^X Y'_n ds+C_1,$$
$$2\operatorname{Re}\left[\mu_1\varphi_1(z'_1)+\mu_2\varphi_2(z'_2)\right]=-\int_0^X X'_n ds+C_2, \tag{9}$$

where $z'_k=z_1^*+\lambda_k^*$, $\lambda_k=(1+1\mu_k)/(1-\mu_k)$, $(k=1,2)$.

The right-hand sides of equations (9) in the total case of boundary conditions (6) and (7) are represented in the form:

$$P_{10}=\frac{R}{2}\left[(-i\lambda_{xy}H+\lambda_yH-P_0)\cdot\sigma+(iC\lambda_{xy}H+\lambda_yH-P_0)\cdot\sigma^{-1}\right],$$
$$P_{20}=\frac{R}{2}\left[(\lambda_{xy}H+iC\lambda_xH+P_0)\cdot\sigma-(iC\lambda_yH+\lambda_{xy}H-P_0)\cdot\sigma^{-1}\right],$$
$$P_{11}=\frac{R}{2}\sum_2^5 d_k\left[(i\lambda_{xy}H+\lambda_yH-P_0)\cdot\sigma^k-(i\lambda_{xy}H-\lambda_yH+P_0)\cdot\sigma^{-k}\right],$$
$$P_{21}=\frac{R}{2}\sum_2^5 d_k\left[-(i\lambda_xH+\lambda_{xy}H-P_0)\cdot\sigma^k-(\lambda_{xy}H-i\lambda_xH-P_0)\cdot\sigma^{-k}\right], \tag{10}$$

Here $P_{1k}=P_{2k}=0$ with $k\geq 2$. In (10) it is considered that

$$X_n^H=-PR\left[C\cos\theta+\varepsilon\sum_{k=2}^5 kd_k\cos k\theta\right],$$
$$Y_n^H=PR\left[\sin\theta-\varepsilon\sum_{k=2}^5 kd_k\sin k\theta\right], \tag{11}$$

The boundary value problem (9) will be solved by using the method of power series of G.N. Savin, and with the help of Cauchy integrals. Solutions to these problems are presented in detail in the works of B. Zhumabaev [13,14]:

The effect of the tunnel σ_x^H, σ_y^H, τ_{xy}^H is established by solving a boundary value problem for plane XOY with a hole, the shape of which is modeled using a mapping function [9].

The stress state of the massifs around the tunnels of the three stress fields at the contour points satisfies the given boundary conditions and can be written as follows [9]:

$$\varphi(\sigma)+\frac{\omega(\sigma)}{\overline{\omega(\sigma)}}\cdot\overline{\varphi'(\sigma)}+\overline{\psi(\sigma)}=i\int_0^s(X_n-iY_n)ds+const,$$
$$\overline{\varphi(\sigma)}+\frac{\overline{\omega(\sigma)}}{\omega'(\sigma)}\cdot\varphi'(\sigma)+\psi(\sigma)=-i\int_0^s(X_n-iY_n)ds+const, \tag{12}$$

According to N.I. Muskhelishvili's method, the boundary condition can be expressed using the tools of the complex variable theory, which leads to the corresponding ratios:

$$(X_n+iY_n)ds=-P(dy-idx)=Pidz,$$
$$(X_n-iY_n)ds=-P(dy+idx)=-Pidz, \tag{13}$$

When a hydrostatic pressure $(-P_0)$ acts on the tunnel contour, the problem of the distribution of stress fields in an isotropic plane with a mine working can be solved using the functions $\varphi(\zeta)$, $\psi(\zeta)$.

This allows us to determine boundary conditions and analyze the behavior of the structure under the influence of external factors [13]:

$$\varphi(\sigma)+\overline{\varphi'(\sigma)}\cdot\left[\frac{\omega(\sigma)}{\overline{\omega'(\sigma)}}\right]+\overline{\psi(\sigma)}=-P_0R\omega(\sigma);$$
$$\overline{\varphi(\sigma)}+\varphi'(\sigma)\cdot\left[\frac{\overline{\omega(\sigma)}}{\omega'(\sigma)}\right]+\psi(\sigma)=-P_0R\overline{\omega(\sigma)}. \tag{14}$$

To determine the functions $\varphi(\zeta),\psi(\zeta)$ from the boundary conditions (12) and (14), which are identical, we combine them as a sum with given constant coefficients

$$N_1=\operatorname{Re}^{i\delta}(S_x+S_y)/2;\ N_2=\operatorname{Re}^{-i\delta}(S_y-S_x+2iS_{xy})/2;$$
$$N_3=\overline{N_1};\ N_4=\overline{N_2};$$

The initial stress state S_x, S_y, S_{xy} was previously defined in [13,14]. The functions $\varphi(\zeta)$, $\psi(\zeta)$ are defined for the exterior of a unit circle, for $q=1$and the contour points of this circle are designated $\zeta=\sigma=e^{i\theta}$ and $\bar{\zeta}=\bar{\sigma}=e^{-i\theta}$.

The Cauchy-type integrals of the boundary conditions in (12) and (14) have the following form [9]:

$$\varphi(\zeta)+G(\zeta)=A_0(\zeta);$$
$$\varphi'(\zeta)\cdot\left[\frac{\omega(\zeta)}{\omega'(\zeta)}\right]+\psi(\zeta)-\overline{G(\zeta)}=B_0(\zeta);$$
$$G(\zeta)=\left[b_3\overline{R_1}+2b_4\overline{R_2}\right]^{2i\delta}\zeta^{-1}+\left[b_4\overline{R_1}\right]^{2i\delta}\zeta^{-2}; \tag{15}$$
$$A_0(\zeta)=\sum_{k=2}^5 ca_k\zeta^{-k};\quad B_0(\zeta)=\sum_{k=2}^5 sb_k\zeta^{-k}.$$

The first equation (14) contains the second-order poles of the boundary condition summand (12). We determine their values by equating them to the coefficients at the same degrees of the variable ζ^{-k}, $(k=1,2)$ of the right and left parts of the first equation. As a result, we have a system of two equations. By adding the adjoint equation to them, we obtain a system of four equations [12,13], where the coefficients on the right side of the system of equations are equal to:

$$MO_0=ca_1;\ MO_1=ca_2;\ MO_2=\overline{ca_1};\ MO_3=\overline{ca_2}.$$

Here:

$$C_1=ca_1+\gamma_1;\ C_2=ca_2+\gamma_2;\ C_3=ca_3;\ C_4=ca_4;$$
$$\gamma_1=(b_3MR_2+2b_4MR_3+3b_5MR_5)e^{2i\delta};$$
$$\gamma_2=(b_4MR_3+2b_5MR_4)e^{2i\delta};$$
$$\gamma_3=b_5MR_3\cdot e^{2i\delta}.$$

The solution to the system of equations in MATHCAD notations is: $MR=M^{-1}\cdot MO$.

$$MR=\begin{pmatrix}-11.105-3.286i\\0.410-0.612i\\0.021+0.721i\\-11.105+3.286i\\0.410+0.612i\\0.021-0.721i\end{pmatrix}$$

Thus, Cauchy-type integrals of the given boundary conditions (12) and (14) are represented by functions $\varphi(\zeta),\psi(\zeta)$ depending on the specific shape of the tunnel cross-section.

As a result of the study, a mathematical model was developed for the stress-strain state around a tunnel with a vaulted cross-section with developed vertical walls at P = -20 MPa. For calculations the numerical values of the initial stress components were taken as:

$$S_x = -10 MPa;\ S_y = 15\ MPa;\ S_{xy} = -10\ MPa;$$

where S_x S_y , S_{xy} - horizontal, vertical and shearing initial stress components. The calculation results are presented in Table 4.

Таблица 4 / Таблица 4

Distribution of stresses around a tunnel with a vaulted cross-section due to water pressure / Распределение напряжений вокруг туннеля сводчатым сечением от напора воды

θ, degrees	$\sigma_\rho(1,\theta)$, MPa	$\sigma_\theta(1,\theta)$, MPa	$\tau_{\rho\theta}$, MPa
-10	-20	27.073	2.576×10^{-14}
0	-20	11.398	4.441×10^{-14}
10	-20	-8.222	2.309×10^{-14}
20	-20	-25.375	3.553×10^{-15}
30	-20	-38.327	6.217×10^{-15}
40	-20	-47.830	-1.510×10^{-14}
50	-20	-51.498	7.994×10^{-15}
60	-20	-45.964	5.329×10^{-15}
70	-20	-36.035	1.066×10^{-15}
80	-20	-27.905	5.329×10^{-15}
90	-20	-22.192	-3.553×10^{-15}
100	-20	-17.217	2.132×10^{-14}
110	-20	-10.388	1.421×10^{-14}
120	-20	4.751	-1.776×10^{-14}
130	-20	52.541	-3.553×10^{-15}
140	-20	120.895	2.842×10^{-14}
150	-20	46.854	8.882×10^{-14}
160	-20	10.104	3.553×10^{-15}
170	-20	92.646	0.000
180	-20	-8.229	1.243×10^{-14}

Fig. 3 [11] shows graphically the distributions of stresses and deformations including combined forces.

The found stress components made it possible to calculate the components of relative deformations using Hooke's Law. The value of Young's modulus E = 1,71 · 10^4 MPa and Poisson's ratio ν = 0,3.

Research results

The model presented for the study and assessment of the stress-strain state of tunnels and workings has a high degree of suitability. The analytical model takes into account the influence of such factors as mountain relief, gravitational and seismic forces, and hydrostatic water pressure on the distribution of stresses and strains.

The distribution of stresses and strains around the tunnel with the combined forces included, changes insignificantly. Fig. 2 shows that tensions occur in the roof and foot of the tunnel, while compression is observed in other areas. The permissible error of the boundary conditions is no more than 10^{-14} and 10^{-15}.

Discussion

All relations required for calculating stress and strain components are algorithmized in MATCHAD notations. Contour stresses arising under the influence of hydrostatic pressure around the tunnel are of a tensile nature, which creates potential threats to weaker rocks. The pressure created by hydrostatic water pressure can vary from 20 MPa to 200 MPa.

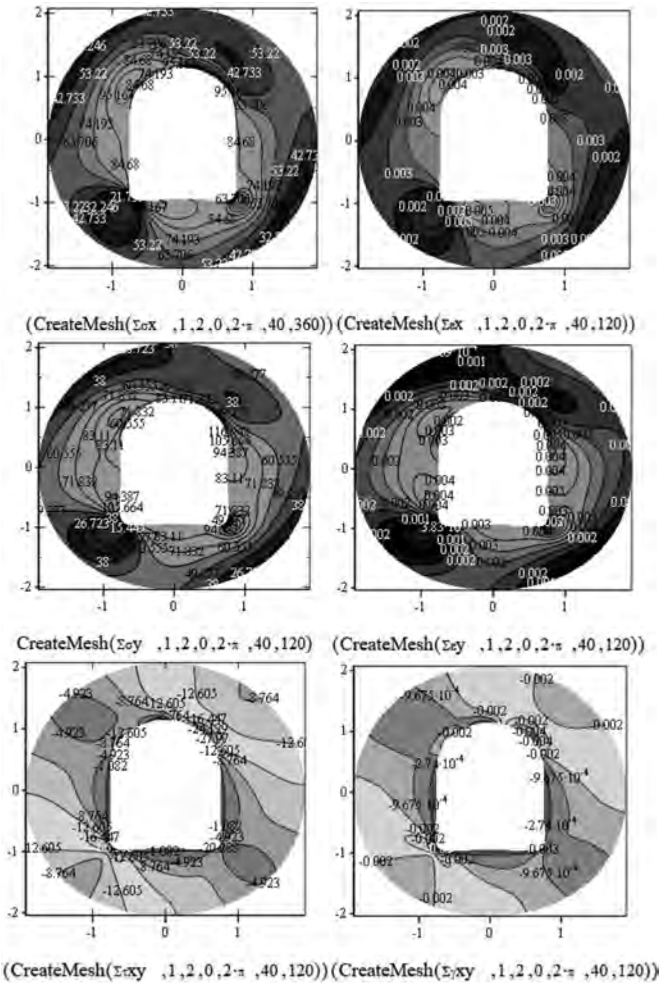

Fig. 3. Isolines of stress distribution (left), relative deformations (right) of the tunnel with the combined forces and hydrostatic pressure of water included /
Рис. 3. Изолинии распределения напряжений (слева), относительных деформаций (справа) туннеля с учетом совместных сил и гидростатического давления воды

Suggestions

The vaulted section of a tunnel with vertical walls allows for efficient load distribution, especially under seismic conditions, and provides high strength and stability [15], making them preferable for designing transport tunnels.

The most important thing when selecting the types and strength of fastening is to take into account the specified values of water pressure P and the maximum values of tensile contour stresses, given in Table 4.

Conclusions

1. The article presents the results of studies of the stress-strain state of rock masses around tunnels located in a dip-layered massif with account of gravitational, seismic forces and hydrostatic water pressure.

2. A stress field σ_x, σ_y, τ_{xy} and σ^H_x, σ^H_y, τ^H_{xy} was developed, arising from the formation of a mine working and the influence of water pressure in an anisotropic massif.

3. New patterns of stress and strain distribution around a tunnel with a vaulted section with vertical walls under the combined action of gravitational and seismic forces are established.

4. It is revealed that the distribution of stress and strain around the tunnel, with account of gravitational and seismic forces, changes insignificantly. It is found that tensions occur in the vault

and foot of the tunnel, while compression is observed in other areas.

5. It is revealed that contour stresses arising under the influence of hydrostatic pressure around the tunnel are of a tensile nature, which creates potential threats to weaker rocks. It was determined that the pressure created by hydrostatic pressure of water can vary from 20 MPa to 200 MPa.

6. The research results can be used in the design of transport tunnels.

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